Bi-layer stress contagion across investment funds: a climate application

Régis Gourdel\textsuperscript{1,2} and Matthias Sydow\textsuperscript{1}

\textsuperscript{1}European Central Bank
\textsuperscript{2}Vienna University of Economics and Business

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Abstract

This paper develops a framework for the short-term modelling of market risk and shock propagation in the investment funds sector, including bi-layer contagion effects through funds’ crossholdings and overlapping exposures. Our application tackles in particular climate risk, with a first-of-its-kind dual view of transition and physical climate risk exposures at the fund level. So far, while fund managers communicate more aggressively on their awareness of climate risk, the risk is still poorly assessed. Our analysis shows that the topology of the fund network matters and that both contagion channels are important in its study. A stress test on the basis of granular short-term transition shocks suggests that the differentiated integration of sustainability information by funds has made network amplification less likely, although first-round losses can be material. On the other hand, there is room for fund managers and regulators to consider physical risk better and mitigate the second round effects it induces as they are less efficiently absorbed by investment funds. Improving transparency and setting relevant industry standards in this context would help mitigate short-term financial stability risks.

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Contact details: regis.gourdel@wu.ac.at
1 Introduction

The search of ways to stress test investment funds has gained new traction due to the growth of the sector and the exposition of its centrality in the financial system. This has driven an increase in the strand of literature dedicated to estimate the risk posed by investment funds to financial stability (Bouveret, 2017; Baranova et al., 2017; Fricke and Fricke, 2020; Gourdel et al., 2019). In parallel, investment funds have been increasingly integrated in more complex models to take into account their interaction with the financial system at large (Farmer et al., 2020). This work integrates this broader effort and the mechanisms developed here are a building block of a larger work on system-wide stress testing realised by the European Central Bank (Sydow et al., 2021).

Due to their increased importance in the financing of non-financial corporations (ECB, 2021), investment funds have positioned themselves as crucial players in the transmission through the financial system of policies to green the economy. Pledges have been made by key members of the industry to integrate better climate risk, either through the use of their voting rights or disinvestment. The sub-sector of ESG and “green” investment funds has grown impressively in the past decade, although its size and technical capacity still do not appear up to the task for steering the financial system to a more climate-vertuous position (IMF, 2021; Schwegler et al., 2021). Moreover, the use of greenwashing by funds to attract investors is a concern for regulatory authorities (Belloni et al., 2020b; Central Bank of Ireland, 2021) and in general for the public interest of taking effective steps toward climate change mitigation. This is also limited so far by an entrenched interest from being historically very invested in carbon-intensive industries as well as a patchy framework when it comes to redirecting investment flows (Mormann, 2020). Thus, Amzallag (2021) finds that European funds still significantly overweight brown firms in their portfolios on aggregate. This suggest that the sector is especially vulnerable to climate shocks. As such, it has also been included in the longer term portfolio losses projections of ECB/ESRB (2021), based on the scenarios developed by the Network for Greening the Financial System (NGFS).

While the market has somewhat adapted to the emergence of climate physical and transition risk, there is evidence that the risk is currently not fully priced. In particular, structural limitations such as policy uncertainty, lack of transparency, insufficiency of existing frameworks to guide green investment, and lack of incentives for fund managers have hampered the adaptation necessary. This explains that the financial sector, and investment funds especially, are still very much exposed to climate risk. Moreover, the presence of greenwashing by funds creates an asymmetry of information between them and their investors, exposing the latter to unexpected shocks related to transition risk. One crucial concern is the possibility of network externalities, where the presence of large risks associated with climate change could lead to systemic failures (Stern and Stiglitz, 2021). In our application, this general systemic risk concern could add to the climate-specific market failure.

The emergence of climate shocks extends the set of questions that are asked in the context of network stress testing. Indeed, in practice most network model applications have focused on two cases so far. The first one was finding how a shock or default of one institution would propagate to other agents. The second was to determine the network amplification of a shock affecting all agents. Climate shocks fall in another category, whereby shocks exhibit a strong heterogeneity in how different groups of agents are impacted. This is particularly true for investment funds, as they often exhibit portfolios that are much more specialised than those of universal banks. Thus, it matters to know whether climate shocks are more likely to be only amplified within a certain subgroup of funds, or transmitted to the rest of the network, and which of these options would actually be best at absorbing damages with the least negative externalities.

Our contribution to the literature is threefold:

(i) We develop a general short-term stress testing framework for investment funds, taking into...
account both contagion channels of inter-fund holdings and common secondary market exposures.

(ii) We build a first-of-its-kind dual climate risk profile for firms and investment funds portfolios, with data on both transition and physical risk exposures.

(iii) We apply our model to a range of plausible shocks, representing climate-related short-term stress.

More precisely, first, our model design allows us to fully capture network externalities. This problem has been tackled in a similar application by Roncoroni et al. (2021), which simulate the reaction to climate transition shocks along theses lines, relying on a bank-fund system. We provide here a framework that is flexible with regard to the input shock and focused on capturing short-term shock amplification from the fund sector. The model unfolds in three steps, starting with a shock on external asset holdings. It is followed by a redemption shock, after which liquidity constrained funds liquidate assets. Our framework can be applied to a variety of narratives, such as extreme weather events, or sudden – and little expected – policy shocks.

Importantly, we show how a new price equilibrium between fund share classes is established consistently with their cross-holdings of fund shares at each step during which their portfolio of external holdings evolves. Thus, we model a shock propagation on two layers: portfolio overlap and direct cross-holdings. The price clearing mechanism is conceptually similar to what is done for network banking models, such that the overall propagation is in line with the literature on stress amplification between banks, occurring through both the inter-bank market and portfolio overlaps.¹

The market granularity of our model allows a design of stress more precise for the time frame where network contagion operates and in particular avoids the distortion of price impact that occurs through coarse aggregations. For that purpose, the static balance-sheet hypothesis that is commonly made, and also present in our work, appears reasonable when compared to most existing longer-term climate stress tests. The use of data from different points in time over the recent year provides an additional security with regard to the static balance-sheet hypothesis, which now comes down to having future medium-term portfolios close to the range of those recently observed. We also allow redemptions based on initial shocks, which are crucial for funds as they are little leveraged and most exposed to liquidity risk from their liability side. Furthermore, this allows us to integrate the key stylized fact that sustainable funds are less prone to pro-cyclical redemptions than their conventional counterparts.

Second, we build a precise ex ante climate risk profile of investment funds using security-level climate risk data combined with fund holdings. Our sample consists of over 23,000 investment funds from the whole world. We find that funds exhibit a strong heterogeneity in their exposure to transition risk, but are more homogeneous with regard to physical risk, presumably in part because this dimension is not integrated in their standard toolkit. Overall, our analysis suggests that the sector taken globally has a high exposure to both types of risk through portfolio holdings when compared to the general economy. This climate risk profile then interacts with the systemicness of the funds at the point of realizing the propagation of climate shocks across the system.

Third, we apply four different kinds of shocks in our model: redemptions by fund investors based on climate portfolio information, a market shock on brown (carbon intensive) assets, a market shock on assets whose issuers are most exposed to physical risk, and the realization of extreme weather events, whose impact is informed by physical risk exposures of firms. In the context of a bumpy adaptation of financial markets to climate challenges, this approach fits better the nature of such shocks. In comparison Battiston et al. (2017) or Roncoroni et al. (2021) are interested in longer

¹Examples include Wiersema et al. (2019) and Poledna et al. (2021), although the counterparty risk transmission channel does not necessarily rely on an Eisenberg and Noe-type clearing mechanism.
term impacts, i.e. over periods of more than 10 years, which would reflect permanent market shifts. Although most economic climate damages will materialize in the long run, we show how short-term shocks are also likely to occur in the process while leading to socially harmful outcomes of their own. Short-term shocks are likely to revert at least partly, but their increasing occurrences could have destabilizing effects on the financial system. The family of network models such as this one is precisely meant to assess the impact of such shocks and their amplification. Therefore, the shocks used in our simulations that are market-induced are calibrated based on the distributions of securities’ past returns.

Our findings suggest that the existing structure of the investment funds sector is relatively robust to climate transition shocks with regard to second round effects, most likely because of the existing specialization into low-carbon industries or integration of ESG standards, which tends to segment the sector, although it does not completely preclude shocks from propagating. On the other hand, there is little resistance to shocks induced by market movements based on physical risk, or shocks from physical climate events themselves. This highlights the importance for the sector to monitor climate adaptation efforts in addition to decarbonization, and to reach a more holistic view of climate risk that integrates its different dimensions.

The rest of the paper is organized as follows: in section 2 we present the model used in our simulations; then, in section 3 we introduce the data used and key stylized facts; in section 4 we detail the design of the shocks applied, and discuss the results before concluding in section 5 with a policy discussion.

2 Core model

The premise of our model is that we operate on a bipartite graph of open-end funds and securities, which we represent in figure 1 as two connected layers. The two types of links that are used correspond to the two transmission channels that come into play. The first type of links are holdings by funds of marketable securities, most commonly stock shares, corporate debt and sovereign bonds. Thus, as securities are marked-to-market, the fund portfolio values depend on how financial markets price these securities, which in turn depends on both the information available to financial agents and the economic soundness of underlying issuers. The mechanism that we describe in 2.6 relies on the common assumption that haircuts on prices applied during fire sales cause a medium-term depletion of asset values, allowing securities to act as intermediaries in a propagation of shocks between funds.

The second link is that of cross-holdings of fund shares in the open-end funds network. The related mechanism is described in 2.2. By comparison to a network of banks, the fund shares cross-holdings share some conceptual similarity with the stock cross-holdings between banks, most notably analysed in Suzuki (2002). However, the comparison is limited because the banks’ shares are traded on the secondary market, with a price determined by demand and supply. Such is also the case of close-end funds, but open-end funds, which are our focus, are different. Indeed, shares of open-end funds are not traded but can simply be bought or redeemed by investors. Moreover, their value is not determined by the market but follows the portfolio value of the issuing fund. In that, there is an automatism in the contagion that is more alike the inter-bank market, which is the focus of most of the literature on contagion risk (Eisenberg and Noe, 2001; Barucca et al., 2020).

The baseline model for the simulations conducted is given in figure 2. As explained in the remainder of this section, the choices made in building the timeline are meant to capture the most likely chain of events and reactions that is compatible with our narrative.

The shocks applied are in fact reflective of the influence of other layers, beyond the internal dynamics of investment funds and the secondary market. Indeed, redemptions would correspond to an additional layers of investors, both institutional and retail, most of them also directly exposed to
the secondary market. Meanwhile, changes in prices in our narrative are based on the interaction with a “real economy layer” and a “climate layer”. First, prices are supposed to be generally reflective of the firm performances from which depend expected future cash flows. Second, interactions between the climate and real economy layers, such as pollution, matter in that market participants anticipating more stringent environmental regulations would then decrease their expectations for future returns of polluting firms. Finally, a shock from the climate layer to the real economy in the form of extreme weather events could cause a number of defaults, which then affects holders of equity or shares issued by defaulted entities.

2.1 Mathematical notation and core model variables

For a matrix $M$, we denote by $M^T$ its transpose. Given an integer $n \geq 1$, let $I_n$ be the identity matrix of size $n \times n$ and $1_n = (1, \ldots, 1)^T \in \mathbb{R}^n$ the vector of ones of size $n$. By abuse of notation, for two vectors $a, b$ in $\mathbb{R}^n$ we write as $\frac{a}{b}$ their element-wise division when there is no ambiguity, meaning $\frac{a}{b} = (a_i/b_i)_{1 \leq i \leq n}$, and $\frac{1}{a} = (1/a_i)_{1 \leq i \leq n}$. Moreover, we denote by $\text{Diag} : \mathbb{R}^n \to \mathbb{R}^{n \times n}$ the diagonal operator. We use bold symbols for matrices to differentiate them from vector columns and scalars in

![Figure 1: Stylized representation of the network agents and key links transmitting economic shocks. Dashed arrows indicate cross-holdings between funds, while solid lines indicate that a security is present in the portfolio of a fund.](image-url)
the following. Finally the operator \( \lor \) denotes the element-wise maximum.

Let \( \mathcal{F} = \{1, \ldots, n\} \) be the index of investment funds in the model, and \( \Omega = \{1, \ldots, m\} \) the index of assets traded on the secondary market. We denote by \( A \in \mathbb{R}^{n \times m} \) the funds’ portfolio matrix of tradable assets, such that for all \( (i, \omega) \in \mathcal{F} \times \Omega \), \( A_{i,\omega} \) is the marked-to-market value that \( i \) holds of security \( \omega \). Thus, the total value of tradable securities of a fund \( i \in \mathcal{F} \) is given by \( \sum_{\omega \in \Omega} A_{i,\omega} \). The second key portfolio matrix is that of redeemable assets, i.e. how much of other funds’ shares are held. It is given by matrix \( R \in \mathbb{R}^{n \times n} \), such that, for \( (i,j) \in \mathcal{F}^2 \), \( i \) holds a value \( R_{i,j} \geq 0 \) of \( j \)’s fund share.\(^2\)

We denote by \( C \in \mathbb{R}^{n} \) the fund-level vector of cash holdings. Moreover, let \( L \in \mathbb{R}^{n} \) the vector of loans from the banking sector to funds, and \( B \in \mathbb{R}^{n} \) the vector of other assets not entering in any of the previous categories. We assume that \( L \) and \( B \) remain constant, i.e. \( \forall t, L(t) = L(0) \), and omit the time variable in their case. The equity of funds, or total net assets (TNA), is represented by the vector \( E \in \mathbb{R}^{n} \). We compute it as the sum of asset holdings, from which bank loans are deducted:

\[
\forall t, \quad E(t) = A(t) \cdot t_m + R(t) \cdot t_m + C(t) + B - L.
\]

The net asset value (NAV), i.e. the price per share, is obtained by dividing \( E \) by the number of shares issued, and therefore in the absence of sales or redemptions evolves proportionally to \( E \).\(^3\)

Moreover, the prices of tradable securities are given by the vector \( P \in \mathbb{R}^{m} \). For tractability we make the assumption that all funds have a positive equity at time 0, i.e. \( \forall i \in \mathcal{F}, E_i(0) > 0 \).\(^4\)

### 2.2 NAV adjustment via direct cross-holdings

We first present the mechanism that describes how the cross-holdings between funds dynamically adapt within the contagion process, as it is used at two points. Although a less commonly used channel than portfolio overlap, it appears also necessary to consider the effect of direct cross-holdings between funds as they constitute a sizeable part of their portfolios. The standard practice for open-end funds is to recompute their NAV daily and to communicate the new value when markets close. Thus, the change in the price of marketable assets impacts the NAV of investment funds that hold them, and in turn affects the amount that their fund investors can redeem. That is true in particular when these investors are also investment funds, which makes the problem more complicated.

For instance, if a fund \( i \) holds shares of fund \( j \) and \( j \) holds shares of \( i \), then the final value of their respective total assets has to take into account this mutual influence. One way to think about it is that, if the value of \( i \) decreases because of a shock on traded assets, then it affects the portfolio of \( j \). Therefore, the TNA of \( j \) decreases, which in turn will impact the portfolio of \( i \), etc.

\(^2\)Note that holdings of tradable securities are allowed to be negative, i.e. funds can short them, while values for cross-holdings are all non-negative. Moreover, when a fund \( j \) is open-end we have \( \forall i, R_{i,j} = 0 \), as shares of \( j \) are counted as tradable securities.

\(^3\)In the case of sales or redemptions, a decoupling occurs such that the NAV does not change but the TNA does.

\(^4\)Cases of negative or zero equity would be likely to reflect data issues when appearing. Moreover, this would mean that no investor has any exposure to such funds, as the values of shares are proportional to the equity. Therefore, we can leave out such cases with little influence on the model.
Thinking sequentially about the issue reflects this daily dynamic, whereby funds would iteratively recompute their portfolio value at fair prices and communicate it to other entities, triggering further accounting adjustments. Such iteration is at the core of Chrétien et al. (2020), where the new information of lower prices creates a shock to portfolios, based solely on financial entities posting new information about their value. Moreover, using that approach, the system will converge to an equilibrium of share prices across the financial system, but may not do so in a finite number of rounds. In contrast, we find directly the full effect of the shock, whereby the change in market prices will establish at once new TNA. The TNA of a fund \(i\) is itself proportional across time to its NAV when the number of shares remains constant. Thus, a new price equilibrium will form.

Let \(t_1, t_2\) be two points in time, with \(t_2 > t_1\), such that marketable assets can change value or be traded by funds from \(t_1\) to \(t_2\), but no fund flows are registered, i.e. there are no redemptions and no fund shares sold to investors. As some funds might default as a result, we denote \(\gamma \subset \mathcal{F}\) a set of funds in default, and \(\bar{\gamma} \in \{0, 1\}^n\) the corresponding solvency indicator such that \(\bar{\gamma}_i = 0\) if \(i \in \gamma\) defaults and \(\bar{\gamma}_i = 1\) otherwise. Moreover, let

\[
X_{\gamma} := I_n - \mathbf{R}(t_1) \cdot \text{Diag}\left(\frac{\bar{\gamma}}{E(t_1)}\right) .
\]

We are then equipped to show how fund cross-holdings will contemporaneously affect the evolution of our system between \(t_1\) and \(t_2\).

**Proposition 1.** If \(X_{\gamma}\) is invertible, the vector of fund TNA at \(t_2\) under a vector \(\gamma\) of defaults is given by

\[
\mathcal{E}(\gamma) = X_{\gamma}^{-1} \cdot (\mathbf{A}(t_2) \cdot \mathbf{t}_m + \mathbf{C}(t_2) + \mathbf{B} - \mathbf{L}) .
\]

The proof of the proposition is given in appendix A.1. More generally, we can dispense with the condition that \(X_{\gamma}\) is invertible,\(^5\) as the only necessary condition is that \(\mathbf{A}(t_2) \cdot \mathbf{t}_m + \mathbf{C}(t_2) + \mathbf{B} - \mathbf{L}\) is in the span of \(X_{\gamma}\). Nonetheless, to make sure that our mechanism is generally well defined, the following proposition establishes that in most common cases \(X_{\gamma}\) will indeed be nonsingular.

**Proposition 2.** Given \(\gamma \in \mathcal{F}\) and cross-holdings \(\mathbf{R}(t_1) \cdot \text{Diag}(\bar{\gamma})\), the matrix \(X_{\gamma}\) is nonsingular if and only if there exists no set of funds \(\mathcal{N} \subset \mathcal{F}\) such that each fund in \(\mathcal{N}\) is fully owned by other funds in \(\mathcal{N}\). This condition is denoted hereafter as property \(H_{\gamma}\).

The proof is given in appendix A.2. A direct consequence is that, if \(X_\emptyset\) is nonsingular then \(X_{\gamma}\) is nonsingular for any \(\gamma \in \mathcal{F}\). As the singularity configuration would make little sense in practice,\(^6\) such cases are rare and likely to reflect data issues. Therefore, we add a step in the data preparation process to remove funds that belong to problematic groups for \(X_\emptyset\).

Given this, our objective is to find a set of defaults such that equation (3) corresponds to a form of equilibrium, based on the criteria that a fund defaults when its equity is negative or null.

**Proposition 3.** If \(H_\emptyset\) is verified, there exists a unique default set \(\gamma^* \subset \mathcal{F}\) such that

\[
\gamma^* = \{ i \in \mathcal{F} : \mathcal{E}_i(\gamma^*) \leq 0 \} .
\]

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\(^5\)Moreover, note that, from the computational perspective, finding \(\mathcal{E}(\gamma)\) from equation (19) is a more optimized operation, that is in particular more efficient than first inverting \(X_{\gamma}\).

\(^6\)Consider that it means the total assets of funds other than fund shares within the group sum to zero. Indeed, if it did not then total equity would exceed the internal share holdings, indicating the presence of external investors. Thus, it means that either funds do not have any assets, or they are leveraged such that their total debt equals the total of their cash and tradable holdings, which is not a sustainable situation.
i.1

1

i.2

i.3

1

i.4

i.5

i.6

0.2

0.12

0.1

i.7

0.3

1

i.8

0.7

i.9

Figure 3: Case of fund system where some funds (in red) break the condition of proposition 2.
A path $i \rightarrow j$ with weight $w$ means that the fund shares of $j$ that $i$ holds account for a proportion $w$ of $j$’s equity. In particular, funds that are problematic all belong to non-trivial strongly connected components of the graph.

The proof relies on a standard fixed-point analysis and is given in appendix A.3. We then denote $E(t_2) = E(\gamma^*)$. In a larger financial system, if there is no additional cyclicity,\(^7\) the assets of other financial institutions can be updated simply by applying the price changes $\frac{P(t_2)}{P(t_1)}$ and $\frac{E(t_2)}{E(t_1)}$ on their tradable and redeemable holdings respectively (the latter being due to a total number of shares assumed constant during that step). In particular, within the system of funds we apply the equation

$$R(t_2) = R(t_1) \cdot \text{Diag} \left( \frac{E(t_2) \wedge 0}{E(t_1)} \right).$$

(4)

In our framework, we apply this mechanism twice. But that needs not be the case and this choice is essentially based on a narrative where different actions happening within the system take a certain time to materialize. For the NAV adjustment, a daily update could in practice deliver the biggest part of the impact in a short time. Only in a system with long chains and cycles of cross-holdings would we expect that the impact after a few days is still significant. To that extent, it seems reasonable to assume that the NAV adjustment happens several times, in parallel to every event that would trigger it. This is still dependent though on assumptions as to how fast the shocks hit and how quickly funds react.

2.3 System amplification potential

The previous results allow some additional analysis prior to any simulation. As we are interested in the impact of small variations in security prices we make the approximation $\gamma^* = 0$, given that most funds have little leverage and would default only under shocks with large magnitudes. Let us denote $Z = X^{-1} \cdot A(t_1)$ and suppose that only asset prices change, i.e. funds do not trade and cash buffers remain unchanged. Then, the evolution of the fund’s tradable holdings will be driven only by the changes in prices:

$$A(t_2) = A(t_1) \cdot \text{Diag} \left( \frac{P(t_2)}{P(t_1)} \right) = A(t_1) \cdot \left[ \text{Diag} \left( \frac{\Delta P}{P(t_1)} \right) + I_n \right],$$

\(^7\)The presence of a feedback loop with other agents, contemporaneous to the one taking place between funds, could invalid equation (3) in a larger financial system.
with \( \Delta P = P(t_2) - P(t_1) \). It follows that the new vector of total tradable holdings is \( A(t_2) \cdot t_n = A(t_1) \cdot \frac{\Delta P}{P(t_1)} + A(t_1) \cdot t_m \). By plugging this in equation (3) we get

\[
E(t_2) = Z \cdot \frac{\Delta P}{P(t_1)} + Z \cdot t_m + \frac{1}{X_0} \cdot (C(t_1) + B - L).
\]

where \( E(t_1) \) can be identified in relation to the special case of equation (3) where marketable assets and cash do not change. From there it follows that \( E(t_2) - E(t_1) = Z \cdot \frac{\Delta P}{P(t_1)} \). Thus, through an ex ante analysis we can identify the combinations of market movements that cause the most contagion from cross-holdings relative to their initial importance. The most important market reactions can be quantified using the norm of \( Z \) (Dahleh et al., 2011), and the corresponding price changes are obtained from the singular value decomposition. Consistent with this, we define the market sensitivity of the fund network as

\[
\text{MS} := \frac{\|Z\|_2}{\|E(t_1)\|_2}, \quad (5)
\]

where normalizing by the norm of the equity allows a comparison across time when computed using data at different dates. This measure has the property to be invariant to the inclusion of securities that are not held by any fund. It is also relatively intuitive as we use unitary shocks, which include singular default events, i.e. the complete write-off of one security. Moreover, it is also possible to quantify how harmful a price shock \( \Delta P \) is for the system, by comparing the realised value \( \|\Delta E\|_2 \) to \( \|Z\|_2/\|\Delta P/P(t_1)\|_2 \).

Additionally, we are interested in quantifying the amplification potential of shocks coming from the cross-holdings. To do so, we do not use general market movements, which can apply in several directions, and we compare the reaction to shocks on single securities, akin to default events. We base this on a comparison between \( Z \) and \( A \), as the latter corresponds to \( Z \) in the case with no cross-holdings. Thus, let \( \text{AMP} \) be the amplification vector such that, for every security \( \omega \),

\[
\text{AMP}_{\omega} := \frac{\sum_i Z_{i,\omega}}{\sum_i A_{i,\omega}} - 1. \quad (6)
\]

To best describe this information, we will rely first on the maximum of this amplification vector, i.e. the security whose write-off propagates the most. Second, we will consider its weighted average, with weights given by \( i_n^T \cdot A \), the vector of holdings’ market values aggregated over the whole sector.

### 2.4 Market shock

In the simulation, funds are exposed to a dual stress materialization: a market shock and a redemption shock. First, the market shock applies to assets present in the funds’ portfolio as their market value changes. A typical example would be common shares of brown firms losing value at the introduction of more stringent environmental regulations, or a firm defaulting because it lost physical capital and income following an extreme weather event. The shock is represented by a vector \( \Lambda \in [-1, \infty)^m \), such that \( \forall \omega, P_\omega(1)/P_\omega(0) = 1 + \Lambda_\omega \). It impacts the funds’ portfolios of tradable assets, with

\[
\forall (i, \omega) \in \mathcal{F} \times \Omega, \quad A_{i,\omega}(1) = A_{i,\omega}(0) \times (1 + \Lambda_\omega). \]
Moreover, \( C(1) = C(0) \) because the market shock had no effect on cash. Given \( A(1) \) and \( C(1) \) we can then compute \( E(1) \) and \( R(1) \) using equations (3) and (4) respectively. Following this step, funds with negative or null equity are removed from the system and considered in default.

### 2.5 Redemption shock

The second shock applied is that of redemptions, which creates a liquidity stress for some funds, although it can be positive net flows for others. The redemption shock comes from investors external to the system of funds. A number of events can push them into moving significant quantities of their assets in a short period of time, resulting in large inflows or outflows for investment funds. It is represented by a vector \( \Psi \in (-1, \infty)^n \) of net flows, which is the ratio of shares sold relative to the previous total. That means \( \Psi_i \) is negative if net redemptions occur for fund \( i \), and positive in the case of net sales.\(^8\) We suppose that redemptions are immediately paid, the shares purchased immediately issued, and the consequences are dealt with in the next step. Thus, we account for it as impacting the cash reserves of funds:

\[
\forall i \in \mathcal{F}, \quad C_i(2) = C_i(1) + \Psi_i \times \left( E_i(1) - \sum_j R_{j,i} \right), \tag{7}
\]

where \( E_i(1) - \sum_j R_{j,i} \) corresponds to how much of \( i \)'s equity is held by external investors. Then, the new TNA vector is given as \( E(2) = E(1) + C(2) - C(1) \). Applying the NAV adjustment mechanism would not be useful here, because fund flows do not affect the NAV \textit{per se}, so \( R(2) = R(1) \). Lastly, \( P(2) = P(1) \) and \( A(2) = A(1) \) as redemptions have no impact on asset prices.

We apply this step after the market shock, which corresponds to a narrative where investors react to changes in the market and do not anticipate them. Nevertheless, the two steps could be easily swapped, whereby the impact from redemptions would be stronger in equation (7). That is because investors would withdraw a share of \( E_i(0) \), i.e. at a point where the posted NAV is still the initial one. Meanwhile, in our configuration they withdraw a share of \( E_i(1) \), which is smaller than \( E_i(0) \) if fund \( i \) was negatively affected, hence a milder liquidity shock. Moreover, proceeding in this order lets open the possibility to calibrate the redemption shock based on the market shock through a flow-performance relationship. In this case, we use the following piecewise linear model:

\[
\forall i \in \mathcal{F}, \quad \Psi_i = \alpha_{i,0} + \alpha_{i,1} \left( \frac{E_i(1)}{E_i(0)} \right)^+ + \alpha_{i,2} \left( \frac{E_i(1)}{E_i(0)} \right)^-, \tag{8}
\]

meaning that investors can have a sensitivity to positive returns that is different from that to negative returns. This is used for open-end funds only, as we take \( \alpha_{i,0} = \alpha_{i,1} = \alpha_{i,2} = 0 \) when \( i \) is a closed-end fund. Such a step has been used in other similar frameworks, such as Mirza et al. (2020), but is generally fully linear or operational on the negative domain only. Thus, our framework is the first to fully embed some convexity in such a model.

Note that from a macroprudential policy point of view, the underlying narrative is different in the case of climate shocks than with a broader economic shock. In the latter, tools such as redemption suspensions usually appear as a valid solution to mitigate spillover effects\(^9\). However, this solution has downsides, such as a reputational risk. This risk relates to the behaviour of investors vis à vis certain funds and might be limited in the case of a recession. When the shock is more differentiated between investment funds, with only some being hardly hit, there could be a higher toll on reputation.

---

\(^8\)The assumption \( \Psi_i > -1 \) ensures that there is no complete run from investors, and therefore no default consequent to this step.

\(^9\)Grill et al. (2021) relate how this was used in the face of the market shock induced by the COVID-19 pandemic. They observe a stronger usage by more vulnerable funds, i.e. leveraged, illiquid, or with little cash holdings.
from taking measures that restrict investors. Thus, there is a shift in the incentives of funds when it comes to using these tools, such that part of the prudential toolbox loses in relevance.

2.6 Fire sales

In order to meet redemptions from investors funds will reduce some of their positions, which will to an extent result in fire sales. The use of cash buffers\[^{10}\] is a first key question as it can change significantly the magnitude of the following step. We know from network stress testing models for banks that, similarly, the extent to which banks are willing to deplete their liquidity coverage ratio has a key influence on subsequent fire sales (Halaj, 2018; Coen et al., 2019). In our baseline simulations we make the hypothesis of fund-level fixed cash targets \((\xi_i)\), as a ratio to total assets. Thus, the total amount that fund \(i\) tries to recover in cash from selling assets is given by \(u = \xi \times (E(2) + D) - C(2)\).

We adopt here a slicing hypothesis, whereby, except for cash, all assets are treated similarly regardless of their differences in liquidity.\[^{11}\] Let \(\hat{v} \in \mathbb{R}^m\) be the vector of volumes sold per assets. We assume that only long positions are sold, and we denote by \(A^+(2)\) the matrix of holdings on long positions.

We get
\[
\hat{v} = \frac{u}{A^+(2)} \cdot \iota_m \cdot A^+(2).
\] (9)

Note that this framework includes cases where funds want to buy assets because they exceed their cash target, i.e. where \(u_i < 0\) for some fund \(i\). However, by initiating purchases in this way there exists a risk that the total of assets held by funds exceeds the existing market caps. Therefore, we limit these proportional purchases so as not to exceed the market caps, and new random holdings are allocated to funds that want to invest further.\[^{12}\] The final vector \(v\) of assets sold is obtained after deducting from \(\hat{v}\) the amounts purchased that we have thus computed.

2.7 Price impact and propagation via portfolio overlap

A large literature exists on fire sales and contagion through portfolio overlap. We build here on existing techniques, with the advantage compared to some previous works of using asset-level information. This avoids the network distortion and overestimation of price impact that can occur when assets are aggregated.

We model the price impact of fire sales based on an exponential specification, in line with Cifuentes et al. (2005),
\[
\frac{P_{\omega}(3)}{P_{\omega}(2)} = \exp \left( -\alpha_{\omega} \times \frac{v_{\omega}}{K_{\omega}(2)} \right)
\] (10)

where \(\alpha_{\omega}\) is the illiquidity coefficient of \(\omega\) and \(K_{\omega}\) is its total market value, which corresponds to the total market capitalisation for stocks or the total amount outstanding for bonds. This price impact function achieves a concavity of the decrease on volume, which is a key feature observed empirically.

As noted in Shleifer and Vishny (1992) and Coval and Stafford (2007), fire sales would materialize when the number of buyers is small compared to sellers, which is more likely to happen when the initial pool of investors for one stock is small. To that regard, fire sales are important to climate stress testing in that shocks are likely to be correlated with the specialization of funds. Indeed, the sector characteristic of companies is an important axis of specialization in the mutual funds universe, but also an important determinant of climate risk exposure as we will show in section 3.3.

\[^{10}\]Although we depart from it, Zeng (2017) provides an important model explaining why funds would try to replenish their cash buffers in the period following redemptions. Chernenko and Sunderam (2016) provides further background regarding the cash management of funds.

\[^{11}\]We have excluded assets that are completely illiquid at the point of splitting between \(A\) and \(B\), so that fire sales can be considered for all assets captured in \(A\).

\[^{12}\]Although this is not designed specifically to thwart the effect of fire sales from other funds, significant purchases might marginally mitigate decreases in prices by this channel.
As a consequence of such correlation, a shock that hits based on climate exposure is likely to lead to sensible fire sales discounts, because many similarly specialized counterparts, who could have bought the stocks in normal times, will be experiencing distress as well.\footnote{On the contrary, trading against constrained funds has been identified as a profitable strategy (Dyakov and Verbeek, 2013). A configuration where funds in our sample benefit from this effect is possible, e.g. if they are sufficiently shielded from negative shocks but have some brown positions that they can expand by taking advantage of browner funds’ distress.}

As tradable assets are marked-to-market, the new matrix of external holdings is given by

\[ A(3) = \left[ I_n - \text{Diag} \left( \frac{u}{A(2) \cdot \iota_m} \right) \right] \cdot A(2) \cdot \text{Diag} \left( \frac{P(3)}{P(2)} \right) . \]

New cash holdings are given by

\[ C(3) = C(2) + \text{Diag} \left( \frac{u}{A(2) \cdot \iota_m} \right) \cdot A(2) \cdot \frac{P(3)}{P(2)} . \]

Moreover, we are able to compute \( E(3) \) and \( R(3) \) using equations (3) and (4) respectively.

The underlying assumption here is that assets have been sold at a discount, which was not taken into account at the point of defining \( u \). Thus, the cash level that is reached is lower than the objective – in the case where changes in prices are all negative. There are broadly two methods that are normally used to deal with this situation. The first one consists in iterating again the whole model, considering the remaining gap as a liquidity shock for the next round. The second, as employed in Sydow et al. (2021), consists in assuming that agents anticipate the price impact and sell more than would normally be needed. In the second case, final volumes are then gradually increased until this iterative process reaches a point where the liquidity demand is satisfied. For our application, we do not opt for any of the two and keep this gap, considering that specifying \( \xi \) in a more conservative way, i.e. with high cash targets, would allow us to achieve the same result.

### 3 Data and measures of exposure to risk

We employ data from a range of different sources. Part of it is investment funds granular data, similar to that used in Sydow et al. (2021). A second part consists of climate-related data, consisting of the same primary data sets than Alogoskoufis et al. (2021).

#### 3.1 Investment fund data

Our primary source of data for investment funds is the Lipper Global Data Feed provided by Refinitiv. The dataset available covers complete holdings of open-end and closed-end investment funds. Holdings data is available monthly and is well populated for the period from mid-2018 to mid-2020. We choose to use primarily values at end-2019, although we run simulations for other months as well.

At the time chosen, the number of funds that are active and hold securities\footnote{More precisely, the filtration applied to the data in order to select funds has four components: funds needs to have at least some tradable holdings (standard stocks or debt securities) or redeemable holdings (the fund shares issued by other funds included), they should have a positive initial equity, this equity cannot be smaller than the total market value of the shares they issued that are reported in the portfolios of other funds, and finally they must not belong to a group that would cause assumption \( H_0 \) to be breached.} in our sample is 23,216. They issue a total of 68,006 distinct fund share classes. The five most important countries of domicile by decreasing order are the USA, Brazil, the UK, Luxembourg, and China (counting primary funds and not share classes). The number of end-of-month holdings of external assets (other than cash or inter-fund holdings) is over 3.3 million, and the number of inter-fund holdings is above 41,000.
As analysed in Cera et al. (2020), funds present a significant connection through overlapping portfolios with their own sector as well as with the banking sector. This is a key motivation for the use of the dynamics presented here in the context of a system-wide stress test. Moreover – when looking at the funds specifically – we can decompose in two the contagion risk \textit{a priori} within the sector: through overlapping portfolios and through direct holdings. To measure the contagion risk from overlapping portfolios we use the cosine measure of portfolio similarity. That is, given \(i\) and \(j\) two investment funds, we define their portfolio overlap as

\[
PO_{i,j} = \frac{1}{\sqrt{\sum_\omega A_{i,\omega}^2} \times \sqrt{\sum_\omega A_{j,\omega}^2}} \times \sum_\omega A_{i,\omega} A_{j,\omega} .
\]

(11)

Similarly, we can define the similarity of \(i\)'s portfolio to that of the sector as a whole:

\[
PO_{i,F} = \frac{1}{\sqrt{\sum_\omega A_{i,\omega}^2} \times \sqrt{\sum_\omega \sum_{j \in F} A_{j,\omega}^2}} \times \sum_\omega \left( A_{i,\omega} \sum_{j \in F} A_{j,\omega} \right) .
\]

(12)

When it comes to the risk coming from cross-holdings, the dynamics are not symmetric, so two measures would be needed. We focus here on the one that quantifies the risk transmission to others from the dynamics of 2.2, i.e. how much the change in the NAV of \(i\) will affect counterparts within the fund sector. We quantify this NAV shock transmission channel as

\[
NST_i = \frac{1}{E_i} \sum_{j \neq i} R_{j,i} .
\]

(13)

such that \(NST_i = 1\) when \(i\) is totally owned by other funds and propagates the entirety of its NAV shock within the system of funds. We combine in figure 4 these two measures of \(PO_{i,F}\) and \(NST_i\). Our data suggests that most funds have a relatively low contagion profile, as the bottom-left corner is very densely populated, which is confirmed by the marginal distributions. Moreover, we observe few big funds that present a high risk on both dimensions, but several big funds appear risky on one of the two aspects.

3.2 Market sensitivity and salient vulnerability

Given the data on funds, and prior to the simulations of section 4, we conduct an \textit{ex ante} analysis of risk using the metrics defined in 2.2. The value added is to quantify the riskiness of the fund network across time in an aggregate manner. First, we plot in figure 5a the market sensitivity of the fund sector, as defined by equation (5), over a range of months presenting sufficient portfolio data. The class of unitary shocks that are considered by this measure can have a significant impact, up to 8% of the sector equity.\footnote{Because of the convexity of the euclidean norm, the stronger shocks picked up by this measure are likely to be close to uniform ones, and therefore exhibit values in line with the uniform stress test that we conduct in 4.1.} Moreover, values exhibit a range of 3%, which indicates that variations in the fund network structure can have implications for systemic risk.

Second, we represent in figure 5b the weighted average of shock amplifications, derived from equation (6). It is also significant, with values around in the range of 10%, and monthly variations that can be large. In general this means that the NAV adjustment mechanism has a sensible impact relative to the original price shocks. Nevertheless, there are important discrepancies in the vector of amplifications, such that we find for some month the maximum amplification to be above 19 times the value of the initial shock, although such extreme cases correspond to securities with relatively little weight.

\[\]
This highlights overall that direct connections also entail their own tail risk. Moreover, the important variations observed between different months suggests that the risk stemming from interconnectedness is very sensitive to the network structure. Although such sensitivity could be difficult to control, this suggests that if a minimization of risk can be achieved, this could happen at a low cost for financial agents involved.

### 3.3 Carbon intensity data

Our first source of data pertaining to the environment is a dataset of firm-level carbon emissions created by Urgentem. It provides data on carbon emissions under scope 1, 2 and 3. For this exercise we use the inferred average carbon intensity on scopes 1 and 2, which is measured in tons of CO$_2$ per million dollar of revenue. The data set used is from October 2020 and covers 46,447 firms. For each of them, observations can be reported for different years. Therefore, we use for every company the most recent reporting available.

We will assume that these scores are indicative of exposure to transition risk. Indeed, highly carbonated supply chains are more likely to become stranded assets or to become less profitable as new climate policies increase the environmental liability of polluting firms$^{16}$. Although there are some limitations to proxying transition risk as carbon emission, it appears as one of its principal factors, e.g. with regards to the introduction or increase of carbon prices. Moreover, in the context of our

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$^{16}$This includes direct carbon pricing, stronger fines in case of breaches to environmental regulations, costlier risk management, etc.
exercise part of the transition risk comes from the market increasingly factoring in climate transition risk. Therefore, what matters is more the information that the market would rely upon rather than the actual long-term determinants of stranded assets and loss of profitability from transition policies. Industry sectors themselves are strong indicators of the emission scores, as we can see from figure 14a, presented in appendix B. Smaller but still significant discrepancies can be observed between countries when aggregating assets at the national level.

Let us denote by $\Omega_U$ the subset of securities where carbon intensity of the issuer is known, and $c: \Omega_U \to \mathbb{R}_+$ the mapping from assets to carbon intensities. Based on this we define a fund-level score called carbon-weighted assets (CWA) such that

$$\forall f \in F, \quad \text{CWA}_i := \frac{\sum_{\omega \in \Omega_U} A_{i,\omega} \times c(\omega)}{\sum_{\omega \in \Omega_U} A_{i,\omega}}.$$  \hspace{1cm} (14)

These values have the advantage of being directly comparable to firm-level carbon intensities. The methodology is very similar to that used by Morningstar to compute its Portfolio Carbon Risk Score, which are then used to identify “Low Carbon” funds. Note that we do not reward funds that are good at screening for green firms, except for the “natural” advantage of firm-level scores. For instance, suppose that $i$ and $j$ invest in the same sector, but $i$ is better at picking green firms while $j$ is uninformed about greenness. Thus, suppose that $i$’s portfolio comprises firms that pollute on average 30% less than those of $j$. Then, the CWA of $i$ will also be 30% smaller than that of $j$, but it would remain high compared to another uninformed fund $k$ investing in a sector that structurally pollutes much less.

The main limitation faced is the cases of funds that only have a small portion of their portfolio covered by Urgentem data. In the charts presented herein we operate a cut-off so as to show only investment funds that have at least a certain ratio of their portfolio scored. Nevertheless, for the purpose of our stress test, we proxy part of the missing holdings using country and sector information.\hspace{1cm} (17)

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\textit{Figure 5:} Exposures of the fund network to price changes.

The market sensitivity measure is based on equation (5) and corresponds to the maximum variation of funds’ equity from shocks of size 1 (in Euclidean norm) relative to the initial equity level. The average amplification is defined by equation (6) and describes how on average the initial shock on one security will be amplified by cross-holdings. Both plots rely on end-of-month data from beginning 2017 to mid-2020.

Source: authors’ computations.
Note that a variety of data providers currently exist, which provide the market with similar information. As discrepancies can exist between them, and carbon accounting methods are complex and imperfect, we do not claim that the measure used in this paper is superior to that developed elsewhere. This uncertainty is also a limitation as to how precisely markets can currently be stirred to mitigate climate change. However, as our aim is to provide a stylized example of how network externalities materialize in reaction to climate shock, we can use this data assuming that it is good enough to be in line with information available at the time of future climate shocks.

From this construction, we can classify funds based on how green their portfolios are. In figure 6 we compare funds when grouped into different buckets (deciles) based on their CWA. Descriptive statistics related to this grouping are presented in the appendix, figure 15a. These figures reflect the expected strength of shock propagation between different groups, where funds within one group are expected to suffer similar initial losses from transition risk. Therefore, it matters if for example an amplification of a transition shock happens mostly between brown funds or if it could propagate to the broader network. Recent work at ESMA has already emphasized this interconnection between brown funds and its implication in case of shocks (Amzallag, 2021).

Figure 6a looks at the portfolio overlap channel, i.e. building on inter-funds common holdings. Its analysis indicates that an amplification within the browner half of funds is indeed plausible. This is consistent with Amzallag (2021), which finds that few funds invest in the same green firms, while brown firms have a large number of investment funds as shareholders. While this is partly confirmed here, with the top decile of green funds having very little overlap with the rest, but the following deciles present a stronger overlap with the rest, and the tenth decile also seems to present little overlap with other groups. Overall, common holdings do not seem likely to propagate shocks from the brownest tier of funds to the greenest one, but the channel as a whole matters in the analysis.

In figure 6b, we see the nominal cross-holdings between the different fund families. Firstly, we see that non-classified funds are the largest holders in general, in part because the group is much larger, and because it contains most funds of funds. Secondly, although no overall pattern emerges, it appears that funds from the first and tenth deciles are the ones most held by others. In that regard, Ammann et al. (2019) find past shifts in flows to be consistent with a reallocation by investors within fund families from least to most sustainable funds, and they suggest that sustainable funds are more likely to be actively marketed within a fund family. Meanwhile, the holdings of fund shares are not taken into account in sustainability indices, such as that by Morningstar’s. In line with this, the appeal of brownest funds would be consistent with a form of greenwashing where part of the flows to more popular sustainable funds would be redirected to brown funds, with no implied penalty in terms of their sustainability ratings.

### 3.4 Physical risk exposure data

The second important input, also at the firm-level, is a dataset created at the ECB from physical risk information provided by Four Twenty Seven. Based on precise locational data and the sector of activity, risk exposure scores are allocated to firms for the different catastrophe types: floods, heat stress, hurricanes and typhoons, sea level rise, water stress, and wildfires. Each score is given as an integer between 0 (least exposed) and 100 (most risky). In figure 14b, in the appendix, are given the average risk exposure of firms for each country. In particular, islands such as the Virgin Islands or Japan occupy the top places due to high idiosyncratic risk induced by their topology. A total of 117

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18Following the framework of Cont and Wagalath (2013), one could suspect that even if green and brown assets were fundamentally uncorrelated or negatively correlated, the distressed selling by funds would induce a positive excess correlation between them.

19Nonetheless, one could explain this discrepancy by the lack of maturity of green financing and we cannot exclude that sustained flows to green funds eventually modify the sub-sector topology. Thus, the appearance of hubs among green securities is a plausible future stability threat.
countries have firms present in the database used.

Our standard indicator of physical risk is an average of the scores on the above mentioned exposures. Similarly to the Urgentem data, we denote by \( \Omega_P \) the subset of securities to which we can attach a physical risk exposure, and \( p: \Omega_P \to [0, 100] \) the mapping from assets to average physical risk exposure. Thus, the fund level exposure to physical risk is given as

\[
\forall i \in F, \quad \text{PR}_i := \frac{\sum_{\omega \in \Omega_P} A_{i,\omega} \times p(\omega)}{\sum_{\omega \in \Omega_P} A_{i,\omega}}.
\]

The dataset provides exposures for a total of 4.3 million firms, by their Orbis identifiers. A total of 128,581 can be mapped to LEI codes. As a large number of assets are not directly covered, we use location-based proxies\(^{20}\), which add roughly almost five times the original number. A decomposition into deciles based on the fund-level PR measure is also used for the related simulations, with corresponding statistics given in 15b. Finally, note that, similarly to the case of carbon intensity, important discrepancies exist between different data providers for physical risk exposure (Hain et al., 2021).

### 3.5 Data matching and fund scores

A key challenge of this exercise is to bring together the data used in a consistent way. In particular, the CWA and PR measures introduced rely on attaching climate-relevant information as well as sectoral, geographical or financial information to our set of asset holdings. We rely primarily on the ISIN

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\(^{20}\)Given an asset \( \omega \) that cannot be mapped to the Four Twenty Seven dataset, we may know its country and zip code from the Register of Institutions and Affiliates Data (RIAD) database. In that case, we proxy its exposure as the average from firms sharing the same zip code, with the condition that there exist at least five of them.
codes of securities, as well as the Legal Entity Identifiers (LEI) of firms. The main resource used is the Centralised Securities Database (CSDB) maintained by the European System of Central Banks. In table 1 we detail the coverage of this matching for carbon intensity data across different categories of assets. One aspect of importance is that we may not want to interpret all matched climate scores equally. For instance, their use in the case of sovereign bonds has a less clear meaning, while they constitute an important part of the sector’s holdings. For that category in particular, the relative lack of exposure data relative to the rest means that they will be less shocked. It appears reasonable for sovereign bonds to be more shielded with regard to short-term shocks, as the transmission channels from climate risk to governments generally consists of longer-term dynamics.\footnote{See Battiston et al. (2019a) for a stress test of these securities.}

<table>
<thead>
<tr>
<th>Number</th>
<th>Market weight (% by value)</th>
<th>With carbon intensity (%)</th>
<th>With physical risk (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agencies</td>
<td>29944</td>
<td>3.01</td>
<td>96.34</td>
</tr>
<tr>
<td>Convertible Bond</td>
<td>12695</td>
<td>0.26</td>
<td>99.96</td>
</tr>
<tr>
<td>Corporate Bond</td>
<td>892122</td>
<td>10.70</td>
<td>99.39</td>
</tr>
<tr>
<td>Equity shares</td>
<td>1947412</td>
<td>69.39</td>
<td>99.95</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>23765</td>
<td>0.33</td>
<td>82.95</td>
</tr>
<tr>
<td>Mortgage-Backed Security</td>
<td>188457</td>
<td>2.66</td>
<td>92.73</td>
</tr>
<tr>
<td>Municipal Bond</td>
<td>56950</td>
<td>0.83</td>
<td>0.49</td>
</tr>
<tr>
<td>Preferred Stock</td>
<td>18059</td>
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<td>99.56</td>
</tr>
<tr>
<td>Sovereign Bond</td>
<td>154317</td>
<td>11.14</td>
<td>0.54</td>
</tr>
<tr>
<td>Supranational</td>
<td>9851</td>
<td>0.15</td>
<td>95.72</td>
</tr>
<tr>
<td>Other</td>
<td>41978</td>
<td>1.12</td>
<td>87.13</td>
</tr>
<tr>
<td>Total</td>
<td>3375550</td>
<td>100.00</td>
<td>92.85</td>
</tr>
</tbody>
</table>

Table 1: Fund holdings covered by Urgentem carbon emission data for end-2019. “Number” corresponds to the total count of securities of the category within the set of holdings, with “% of total amount” indicating the weight of each category by market value. “% scored” indicates the ratio with score from the Urgentem dataset within each category (by the number of securities and not weighted by market values).

Additional information on securities is obtained through CSDB. For instance, the total market valuation in euro of securities $K$ is imported directly from the database. Where not directly available we compute it as the product of the price and the number of outstanding securities, or we propagate the value used in the previous period.

The key value added and novelty of this is that we can decompose climate risk along its transition and physical dimensions, based on our different indicators. Although the matching is not complete, this can be first visualised at the firm level. We show in figure 7 the joint distribution of risk obtained from these two rating methods, on the universe of firms with complete information. The firms included are all those with information available, and not only the ones issuing on the market. The next step is observing this at the fund level. As funds have portfolios that are at least somewhat diversified we observed less outliers than when looking at firms directly. Crucially, we can notice that the point of higher concentration for funds is closer to the upper-right corner. This suggests that investment funds have a bias towards firms that are relatively more risky, both from the transition and physical perspective.

More precisely, from an analysis of transition risk alone, we find that in 2019 the carbon intensity of the funds’ assets were on average in excess by 6.9% compared to our unweighted sample of firms with data (excluding those proxied). This unweighted benchmark corresponds to the $x$-axis in the left-hand side of figure 7. In that case, the bias of funds is actually less strong than the one of the market in general (based on the total market valuations of assets), which is in excess of 22.4% on
Joint risk distributions for firms of our sample and investment funds. Both plots are given as bi-dimensional histograms, with the colour scale indicative of the number of entities of each tile. Left panel: data for real economy firms included in our sample. The x axis corresponds to the carbon intensity measure $c(\omega)$, in log scale, while the y axis is indicative of the measure $p(\omega)$ of physical risk exposure. The joint representation is obtained after matching both types of data on the LEI codes. Right panel: data at the fund level after taking weighted-averages of portfolios. The x axis corresponds to the carbon intensity measure CWA, in log scale, while the y axis is indicative of the measure PR of physical risk exposure. Source: Refinitiv, Urgentem, Four Twenty Seven and ECB calculations.

Figure 7: Joint risk distributions for firms of our sample and investment funds.

average over the same period. This means that the funds included in our sample already correct for part of market excesses when defining their portfolios. However, that is insufficient to reach an actually “virtuous” level of carbon intensity, which would have to be below our general benchmark. A visualisation for the related time series is available in the appendix, figure 16a.

When looking at physical risk, the excess exposure of funds relative to our general sample of firms is 34.2% in 2019 on average, compared to a general market excess of 27.4%. This means that a bias toward riskier assets is built up on two levels. First, firms with higher physical risk exposures tend to issue more on the market. Second, investment funds tend to invest more heavily in the riskier firms, even relative to securities available. The latter step is presumably not carried out on purpose but an externality of investment strategies that present a positive correlation with physical risk. For instance, the lack of investment in adaptation might be rewarded, as it would be financially advantageous in the very short-term. The related chart is available in the appendix, figure 16b.

Overall, there seems to be a potential for funds to pivot their portfolios towards firms that are better prepared for climate change impacts, and to a lesser extent less polluting. Furthermore, even a relatively safe portfolio does not prevent the effect of contagion coming from more exposed funds in the short term, and over a longer horizon this exposure is likely to materialize as significant losses if markets and portfolios do not evolve sufficiently.

4 Results

Given a system of investment funds built from our data, we estimate its reaction to shocks following a propagation mechanism as described in section 2. The shocks are defined in this framework, i.e. as an initial change in prices $\Lambda$ and initial fund flows $\Psi$.

First, in section 4.1 we benchmark the system of funds against uniform shocks, such as used in a

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22The market bias toward more carbon intensive firms is already known in the literature and affects other institutions such as central banks (Matikainen et al., 2017).
number of other papers. Second, we design several plausible shocks based on climate data features that translate into risk for portfolio exposures. This allows a finer event analysis than exercises that rely on a pure redemption shock, often not using very granularly the funds’ features (ESMA, 2019).

4.1 Sensitivity analysis from uniform shocks

Another way to assess the functioning of our model and simulation framework is to observe its response when faced with uniform shocks from the market. That means we apply a series of shocks $\lambda \in [0, \frac{1}{2}]$ such that $A(1) = (1 - \lambda) \cdot A(0)$. Coarse shocks such as a uniform change in the yields of bonds are standard in the stress testing literature where the focus is on the model and not on the scenario.\textsuperscript{23} Using it allows us to compare more easily our results. This is done in combination with redemption reactions based on a linear flow-performance relationship. Different values for the flow-performance driving coefficients are tested. Results are decomposed into the following components:

a) direct gains and losses from the initial market shock;
b) gains and losses from the first round NAV adjustment on cross-holdings;
c) outflows from the performance-based redemption shock;
d) gains and losses from the price impact;
e) gains and losses from the second round NAV adjustment on cross-holdings.

We reference as indirect gains the components b, d, and e. For each of these components we take the sum over all funds covered in our analysis. The output using our reference month of December 2019 is given in figure 8. Note that the first plot is linear because it is equal to $-\lambda \sum_i \sum_\omega A_{i,\omega}$, and the second one as well, since based on the results from section 2.3 we can isolate the effect of cross-holdings revaluation as $-\lambda \sum_i \sum_\omega (Z_{i,\omega} - A_{i,\omega})$. This last term can also be written as $-\lambda \sum_\omega (AMP_{\omega} \sum_i A_{i,\omega})$. Thus, the average amplification that is represented in figure 5b gives the ratio in case of a uniform market shock between the direct gains and the ones from cross-holdings adjustments.

We can observe that the price impact has a small positive effect in the case with no flow-performance mechanism. Indeed, this corresponds to a case where funds with mostly long holdings see the value of their portfolio decrease, while the cash and equivalent maintains the same value. Thus, cash rises in proportion of total holdings, and as no liquidity shock follows funds can buy securities to maintain the same cash ratio. Furthermore, note that the negative effect of the price impact disappears for large negative shocks in our framework in the case of an important flow-performance relationship.\textsuperscript{24} This is because the negative price impact effects are limited by the ratio of external investors that can redeem, and the share of tradable security over the whole portfolio that can be sold or affected by the trades of others. Thus, the positive effect takes over after some point due to the funds with short positions and those with a small external investor base where the flow-performance effect is small.

It comes that the average amplification as defined above also provides a benchmark to compare the effect of other shocks on the period between times 0 and 1. Yet, this does not provide for a comparison over the whole simulation. To this end, we will use uniform shocks as the benchmark for simulations with heterogeneous market shocks. Given a general price shock $\Lambda$ as used in section 2.4, define the direct gains function $g_1: \Lambda \mapsto t_n^T \cdot A \cdot \Lambda$. Then, the uniform shock that results in the same direct gains as $\Lambda$ is given by

$$u(\Lambda) := \left( \frac{g_1(\Lambda)}{t_n^T \cdot A \cdot t_m} \right) t_m,$$

\text{\textsuperscript{16}} i.e. such as $g_1(\Lambda) = g_1(u(\Lambda))$. Then, we use this shock as a base to qualify the indirect gains $g_2(\Lambda)$ that follow market shock $\Lambda$, which is given by the sum of components b, d and e above, and

\textsuperscript{23}Some scenarios such as the ones used by the EBA also feature shocks on securities with little granularity, i.e. defined at the country level.

\textsuperscript{24}The largest flow-performance coefficient of 0.8 that is used here is larger than our calibrated coefficients for negative returns given in 4.3, but smaller than the ones used for positive returns.
IS(Λ) := \frac{g_2(Λ)}{g_2(μ(Λ))} the indirect severity indicator. We will then say that Λ exhibits severe indirect effects if IS(Λ) > 1, and mild indirect effects if IS(Λ) < 1.

Moreover, this exercise proves that second-round effects exhibit a strong sensitivity to the flow-performance parameter. While the magnitude of the reaction is fading in the last steps, it remains economically sizeable. Moreover, in the context of a larger stress test, with all financial agents accounted for, losses would be larger as they account for the funds’ total reduction in equity and the effect of price changes on all securities present in the market.

4.2 Transition shock with score-based redemptions only

In a first stage we study the effect of a shock that affects funds through net flows only, i.e. with no initial market shock. This corresponds to a narrative of a policy shock designed specifically to benefit green investment intermediaries, e.g. an extended official labelling of funds based on carbon intensity, or a change by market data providers in the information on carbon intensity that is given to investors. This could also reflect more general policies such that the reaction from fund investors trumps the effect on security prices.

In figure 9 we show that a climate policy announcement such as the Paris Agreement can be correlated to unusual flows, with in particular an effect on funds that classify as socially responsible investment (SRI) or environmental, social, and governance (ESG) that experienced significantly higher flows than their average. The effect observed is in line with Monasterolo and De Angelis (2020), which finds that the Paris Agreement has made green finance more appealing without penalizing significantly carbon-intensive assets. Moreover, Ceccarelli et al. (2020) and Hartzmark and Sussman (2019) show that improved information on sustainability and carbon-related risk also has a potential
to affect flows, favouring most sustainable funds at the expense of the least ones. Thus, existing evidence points to the fact that investors react to the available information and internalize the expected consequences from transition policies.

\[ \Psi^i \] carbon-weighted assets by fund from equation (14), and the percentage changes correspond to values ordered from green to brown: 1%, 0.5%, 0%, -1%, and -5%. The distribution itself is based on the previously described, we suppose the following net flows on the different quintiles of funds when based on the portfolio carbon rating of the funds. More precisely, to replicate features of the shocks of green funds to react to positive shocks.

The existence of a strong positive inflow shock also matters here and does not mechanically result in a desirable outcome. Coval and Stafford (2007) find that extreme inflows can be costly for mutual funds as they tend to increase their existing positions, creating additional price pressure on the holdings common with their peers. In the case of climate shocks, while steady positive flows could ease the financing of the green economy, abnormal inflows to green funds could be less useful, providing instead trading opportunities to outsiders. To that extent, there is a social cost to these sudden shocks, even in their positive materialisation. Existing barriers can reinforce that effect in the case of green funds. In particular, the information acquisition related to screening for green assets can be costly or take long to proceed. Additional monitoring for green features can add to the burden of these funds, when compared to more traditional funds that do not have this dimension. Thus, portfolio extensions are difficult to carry in the short term.\textsuperscript{25} This, in turn, can be alleviated via a greater transparency: cheaper or more readily available information on green factors. Improved regulation regarding carbon disclosure related to financial assets could especially enhance the ability of green funds to react to positive shocks.

Results for this first exercise are given in figure 10, that is supposing that redemptions occur based on the portfolio carbon rating of the funds. More precisely, to replicate features of the shocks previously described, we suppose the following net flows on the different quintiles of funds when ordered from green to brown: 1%, 0.5%, 0%, -1%, and -5%. The distribution itself is based on the carbon-weighted assets by fund from equation (14), and the percentage changes correspond to values \( \Psi^i \) as defined in section 2.5. This results in moderate returns overall, with a stronger amplification of the positive shock through price impact. In line with results obtained in section 4.1 we find that second round gains from cross-holdings are of smaller magnitude than the gains from price impact, except for non-classified funds as they do not suffer from self-inflicted price impact.

\textsuperscript{25}The fire sales step introduced in 2.6 is consistent with it as the extension of portfolios take priority over the purchase of new assets.
4.3 Transition shock affecting asset prices

In a second scenario, we consider a shock on market prices driven by transition risk, and reinforced by redemptions through a flow-performance response. This supposes the materialization of a transition shock such that investors, for example retail-driven, disinvest from highly polluting sectors and from funds whose carbon intensity is high. This could also reflect for instance an unexpected policy shock.

The analysis of transition shocks of the sort has been conducted for instance by Belloni et al. (2020a), who simulate the impact of short-term credit downgrades from sector-level carbon exposure on banks.

We elaborate a deterministic price shock on securities based on their corresponding carbon intensities, such that brown securities depreciate while the price of greener securities increases. To that end, we use CSDB price data to fit Student-t distributions to all series of monthly prices where securities have more than two year of data available. From this fitting exercise, for every security \( \omega \) where data on prices is available we derive a quantile function \( Q_\omega : [0, 1] \to \mathbb{R} \), modified so as to clip the outcome\(^26\) to the interval \([-1, 1]\). Then, if carbon intensity information is available for \( \omega \), the shock to its price is computed as

\[
\Lambda_\omega = Q_\omega \left( \chi_0 + \frac{1}{1 + \chi_1 + c(\omega)/\chi_2} \right)
\]

with \( \chi_0 \in [0, 1), \chi_1 > \frac{-1}{1-\chi_0} - 1 \), and \( \chi_2 > 0 \) are parameters of the quantile mapping. Details of the resulting shock are represented in the appendix, figure 17a. In the case of securities where carbon intensity is available but not the price distributions, the shock is proxied from securities with the closest characteristics.

Bolton and Kacperczyk (2020) find that only the level of emissions matters in the observed carbon premium, and that this is based on scope 1 emissions only. This is supported by Ehlers et al. (2021), which find that transition risk corresponding to scope 2 emissions are not appropriately factored by banks in spite of their relevance. Therefore, by using a measure of scope 2 emissions we depart from

\(^26\)On the negative side, the limit chosen allows for complete defaults while preventing prices to go negative, and as we do not model large positive shock the upper bound prevents large returns that could be driven by data imperfections. Moreover, this roughly corresponds to the 1% highest absolute monthly returns.
the already observed dynamics, toward a narrative where markets start reacting to more exhaustive information about their carbon contribution than what is currently the case. Thus, the shock is consistent with a market that corrects for its previous lack of data exploitation. Moreover, Bessec and Fouquau (2020) observes from textual analysis the influence of a green sentiment on the financial market, confirming that a reaction is often to be expected relative to climate news.

In combination with this shock we use the flow-performance relationship described in 8. We make the conservative hypothesis of null flows in the absence of returns, i.e. \( \forall i, \alpha_{i,0} = 0 \) and the sensitivities to return are calibrated based on the flow-performance study of Renneboog et al. (2011). More specifically their results isolate funds with green screenings from the rest. For a conventional open-end fund \( i \) (meaning not green), its sensitivity coefficients are \( \alpha_{i,1} = 1.557 \) and \( \alpha_{i,2} = 0.553 \). Meanwhile, if \( i \) is green then \( \alpha_{i,1} = 2.316 \) and \( \alpha_{i,2} = 0.546 \). In particular, this specification reflects for all funds the convex character of the flow-performance relationship, which is observed across most studies.

Results for this exercise are provided in figure 11. Direct gains are overall consistent with the categorisation of funds, as the browner ones lose more. The subsequent investment flows are in line with it, and demonstrate the convexity of the relation, as net gains entail a relatively stronger reaction than negative ones. Indirect gains are an order of magnitude lower than direct ones, hence an overall moderate amplification. We see that gains on cross holdings are negative for all categories of funds in the first round, which confirms the possibility of a contagion through this channel, and this is true across most other months with data. Market gains (from price impact) are broadly in line with the sign of the initial shocks, except for the greenest decile, which suffers from negative spillovers.

**Figure 11**: Results for a scenario with carbon-based market shock.

Funds are ordered by green deciles, i.e. group 1 corresponds to those whose CWA is in the 10% lowest. The group NC corresponds to funds that are not classified from their CWA because the part of their portfolio with carbon intensity information available is too small.

Left panel: direct gains of funds on their portfolio of tradable holdings as entailed by the initial price shock. The aggregation at the decile level is obtained by summing total gains in a decile, and dividing by the sum of initial assets. Central panel: net flows to each decile of funds due to the flow-performance investor reaction. Right panel: indirect gains happening as a reaction to the initial shock and the subsequent flow-performance investment.

Source: author’s calculations.

Crucially, the flow to green funds are very strong in case of positive returns, thus the intermediary reaction of investors will reinforce positive spillovers in that case. Therefore, embedding the convex feature of the flow-performance relationship appears key in a setup where we deal with shocks that are complex in nature, i.e. more sophisticated than the uniform stressing approach. Moreover, the effect from the second round of changes on cross-holdings is again significant for non-classified funds only, which also present a large reaction to the first round cross-holdings changes. This can be explained
by the fact that many funds are non-classified because they are funds of funds – or at least a large part of their portfolio is made up of other funds – so that by nature they are more affected by this step.

We find $\text{IS}(\Lambda) = 0.66$, meaning that it is overall less severe than a uniform shock of a similar first order impact. The series of values for the relative severity of this exercise at different times is given in the appendix, figure 18a. We see, in line with results from 3.2 that the indirect severity has some changes, but it is always clearly below 1. Moreover, the trend over the sample period is overall decreasing, which may reflect an increased specialization of funds with regard to carbon intensity metrics.

### 4.4 Results from physical risks market reaction

As a first result related to physical risk we suppose an improved pricing by the market, such that the price of securities more exposed to physical risk decreases, while some securities at the other end see positive returns. Similarly to the previous market shock, we define a heterogeneous shock between assets based on the physical risk exposure:

$$
\Lambda_\omega = Q_\omega \left( 1 - \kappa_0 - (1 - \kappa_1) \times \frac{(p(\omega) - \min_{\varepsilon} p(\varepsilon))}{\max_{\varepsilon} p(\varepsilon)} \right)^{\kappa_2},
$$

(18)

with $\kappa_0 \in (0, 1)$, $\kappa_1 \in (\kappa_0, 1]$ and $\kappa_2 \in \mathbb{R}^+$. Details of the resulting shock are represented in the appendix, figure 17b. We use the same flow-performance calibration as in section 4.3.

Results for this exercise are provided in figure 12. The first key fact is that the coverage of physical exposures is worse than for carbon intensity. Therefore, less securities are stressed, explaining that losses appear moderate, although the ones that are stressed suffer shocks of the same magnitude as in the transition risk market shock. From the representation of direct gains, we can first observe that the shock is more equally distributed across funds from different segments compared to the transition market shock, to the point that funds in the first two deciles incur more losses than those of deciles 3 to 7 in relative terms.

This counter-intuitive allocation of the shock has several explanations. First, this sheds light on the fact that funds do not organise or define their portfolio as a function of exposure to physical risks currently. This is to the contrary of carbon exposure, such that a shock based on physical risk is more similar to a shock randomly distributed. Therefore, the difference in \textit{ex ante} physical risk exposures of funds is small (see table 15b in the appendix). Second, the volatility of assets matter. For instance, a fund holding equity securities of mildly exposed firms can lose more than a fund holding bond securities of very exposed firms. Lastly, a bias in our representation comes from the fact that the shock is computed over the total assets of the funds, while the shock hits only part of them. Thus, a fund that holds a large number of securities with no exposure information (like sovereign bonds, see table 1), will amortize the shock better and exhibit a smaller relative loss, even if its corporate holdings are very exposed to physical risk.

The net flows observed are all negative, following closely the the initial impact as expected. Then, second round effects seem broadly in line with the initial shock, with the notable fact that non-classified funds that had little direct losses are the ones losing most indirectly. We get $\text{IS}(\Lambda) = 0.96$, also meaning that such a shock causes less indirect damages than a flat out crisis, but is close to it because as explained above there is a lesser correlation with the network structure. The full time series is provided in the appendix, figure 18b, where we observe that indirect severity for this exercise is in the window of 91% to 97%, which is higher than that from the transition risk market shock but always below 1.

This lesser absorbance of the shock by the system can be understood in the framework given by
FIGURE 12: Results for a scenario with a market shock based on physical risk exposure.
Funds are grouped by deciles based on their prior risk exposure, i.e. group 1 is the category whose prior exposure to physical risk is in the lowest 10%. The non-classified (NC) category contains all funds for which less than 20% of their portfolio has a known physical exposure.
Left panel: direct gains of funds on their portfolio of tradable holdings as entailed by the initial price shock. The aggregation at the decile level is obtained by summing total gains in a decile, and dividing by the sum of initial assets. Central panel: net flows to each decile of funds due to the flow-performance investor reaction. Right panel: indirect gains happening as a reaction to the initial shock and the subsequent flow-performance investment.
Therefore, as securities with unknown risk are not stressed, the NC category also suffers benign direct losses.

Pástor et al. (2020), whereby the introduction of ESG and climate risk concerns imply a four-fund separation for investors: a riskless asset, a market portfolio, an ESG weighted portfolio, and a climate risk weighted portfolio. While ESG and climate risks can be correlated in their framework, they can also be more orthogonal if we take the latter as reflective of physical exposure. In our case, the results suggest that while a carbon-weighted portfolio – akin to an ESG one – could be a structuring element of our system, there is no such thing as a common climate risk portfolio that would be used to hedge physical risk. This could be attributed to both a weaker investor preferences and to a lack of information at the firm level.

4.5 Results from physical risk damage materialisations
Beyond their immediate impact, physical shocks from extreme weather events affect the economy through channels such as income destruction, credit shrink or defaults (Schydlowsky, 2020; Monasterolo, 2020). Under this scenario, the shock materialises in two steps. First, a connected series of extreme weather events occur. This causes a loss of profitability or complete default for firms as they lose part of their physical assets or are impaired in their operations, because their immediate environment or segments of their supply chain are affected. Second, the asset prices decline based on this event.

Investment funds are in general not likely to be the sector most affected by such shocks in comparison to banks or insurance undertakers that would lose on their loans to SMEs or would have to directly compensate victims respectively. However, because funds do not seem currently to integrate that dimension in their portfolio choices, a shock from a physical climate event large enough to cause market turmoil could hit them hard. The literature for stress testing short-term climatic events is scarce thus far, but studies such as Mandel et al. (2021) in the case of floods show that shocks could be propagated and amplified by the domestic and international banking systems, especially in the future in the case of insufficient adaptation. Therefore, this could push the financial system in a stressed position where funds would be hit too and other sectors are even more stretched and cannot absorb it.
The simulations performed are divided into the different kind of physical risks considered: floods, heat stress, hurricanes/typhoons, water stress, and wildfires. For each run, an issuer-level shock is determined randomly such that $-\Lambda_\omega = (x_{\xi(\omega)} \times y_\omega) \wedge 1$ where $\xi(\omega)$ is the country of the issuer of $\omega$, $x_{\xi(\omega)} \sim \mathcal{U}([0, 1])$, and $y_\omega \sim \mathcal{E}(p^k(\omega)/100)$ is an issuer level shock given by an exponential distribution whose mean is the known exposure $p^k(\omega)$ to a source $k$ of physical damage. Thus, we perform a series of Monte Carlo simulations such that each run corresponds to a scenario with differentiated shocks between countries, and differences between issuers in a country are informed by the physical exposure to that type of shock. The flow-performance calibration of section 4.3 is again used on top.

Results for this exercise are given in figure 13. We observe that shocks defined in this way can have material effects, with more than 20% of the sector’s equity wiped out in the most extreme cases. Crucially, the changes in equity for the total investment fund sector exhibit different patterns depending on the type of shock that is used. While the difference between shock types is very dependent on the metric initially used in the exposure assessment, and therefore has little comparative power, the spread of values within each shock type is more indicative. Thus, we see that wildfire, water stress and heat stress are the ones where tail events can damage investment funds the most.

**Figure 13:** Results for a range of physical shock based on firms’ exposures. The box plots and scatter plots correspond for each shock category to a total of 200 Monte Carlo simulations, produced from shocks generated based on exposures to physical risks. Left panel: total equity change of the sector, i.e. after summing over all funds, in the course of the simulation. Right panel: indirect severity of the simulations, i.e. the level of indirect damages against that of a similar uniform shock, as defined in 4.1.

## 5 Conclusion and policy discussion

In this paper, we have developed a model to assess the amplification of financial shocks happening within the sector of investment funds on an international scale. The method presented innovates in the scope of the sector that is used to better characterise shock transmission, in particular including in a consistent manner all open-end and closed-end investment funds globally. Our application to climate risks suggests that the short-term type of shocks that can be linked to climate present a limited risk of amplification within the fund sector alone. Our data analysis, however, confirms the potential for important longer-term losses, and that the short-term amplification may be reinforced in the interaction with other financial sectors.

Changes in regulation with the aim to reduce systemic risks from network amplifications have been proposed before for banks (Poledna and Thurner, 2016). Nonetheless, doing so for funds appears more challenging given the lower supervisory scrutiny and regulation. Moreover, accounting for the network effect exposed here would be best done within a broader contagion framework such as in Sydow et al. (2021). Still, measures for amplification risks such as the ones introduced in 2.3 could
be used further, and are compatible with other data providers.

In our application to climate shock, we covered different types of shocks that were likely to materialize in a short time frame and be subject to amplification effects. The coarse design of each shock type would need to be improved by further research, so as to provide a more exhaustive exploration of shocks with identified likelihoods. More work has been done in that regard so far in the case of transition risk (Battiston et al., 2017; Battiston et al., 2019b; Reinders et al., 2020), but a lot remains to clarify for both transition and physical risk. In particular, although the method to derive shocks from the data ought to be refined, note that a much more important discussion is the standardization and correctness of the data used as an input (Schnabel, 2021). Indeed, as massive discrepancies between providers exist for both categories of climate data used, the variation coming from initial measurement errors trumps subsequent model uncertainties.

With regard to the climate transition more specifically, most recent reports on the evolution of the sector show that funds labelled ESG or SRI are now prominent drivers of the industry growth. This growth of green funds, in its steady aspect alone has raised the concern that green stocks are overvalued because they are part of a pool of assets that is too small. The suggestion often derived from the situation is the need for a stronger governmental action in certifying certain industries as climate friendly or easing the development of green firms. Some of the transition shocks that can plausibly happen in our time frame relate to this currently existing gap. Indeed, as environmental practices and reporting are not sufficiently streamlined, markets suffer from uncertainty in their estimates of climate risk, even in the optimistic case where they would be willing and capable to factor it in.27 Meanwhile, the greenness assessment that is provided to some market participants cannot be assumed to perfectly reflect the impact of future transition policies, in particular when commitments to the latter are questionable.

Furthermore, as non-pecuniary incentives have been identified as drivers of flows, a convergence of the standards applied by regulators and market data providers would provide more clarity. Then, a more holistic approach could be preferred to simply improving the existing portfolio assessments discussed above. Consider for instance that funds holding brown assets often fall short of their green commitment in that they fail to use their voting rights in a climate-friendly way. However, there is currently little incentive for them to do so apart from media scrutiny. Thus, some key actors may need to engage in the debate on the efficiency of disinvestment or internal activism so as to best incentivize funds in a manner that profits climate change mitigation. This would also include a better accounting of cross-fund holdings. Indeed, as investment funds by themselves are not carbon intensive, they often tend to bring down the carbon intensity of portfolios. This currently constitutes a form of greenwashing as some funds can invest in other funds browner than them without any negative consequences on their ratings. As we have seen, this particular channel can also reinforce risk as funds that should be greener can then suffer when carbon intensive assets lose value.

Thus, an assessment of financial agents’ climate commitments is likely to suffer subsequent perturbations through regulatory improvements if its current criteria do not optimally assess climate-friendly behaviours. This combined with the other sources of uncertainty exposed makes the market still quite prone to a form of “climate volatility”. In that respect, our findings support the calls to judge financial institution by the carbon intensity of their portfolios instead of that from their own operations. In the medium term a more multi-dimensional approach might at least mitigate the relative impact of uncertainty from individual sources. Although the risk of systemic events appears limited in our current framework, there is a financial risk interest in reducing the risk from misunderstood climate exposures, whether through regulatory standards or pushing for best practices in the industry (see for instance Popescu et al., 2021). Additionally, this identification brings the complementary question

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27Consider for instance the relative absence of impact assessment with regard to biodiversity (NGFS, 2021).
as to whether the network structures that are best suited to weather climate shocks are also optimal against more standards risks.
Bibliography


Battiston, Stefano, Antoine Mandel, and Irene Monasterolo (2019b). “CLIMAFIN handbook: pricing forward-looking climate risks under uncertainty”. In: *Available at SSRN 3476586*.


Bessec, Marie and Julien Fouquau (2020). “Green Sentiment in Financial Markets: A Global Warning”. In: *Available at SSRN 3710489*.


A NAV adjustment details

A.1 Proof of proposition 1

We prove below the first analytical result, i.e. the form of the equilibrium NAV after market changes given the non-singularity of $X_{\gamma}$ defined by equation (2). The case of non-singularity itself is established in the following sub-section.

Proof. Let us denote by $\pi \in \mathbb{R}^n$ the vector of funds NAVs. First, note that in the absence of any fund flows, the NAVs will change from $\pi(t_1)$ to $\pi(t_2)$, with $\pi(t_2) = \text{Diag}(\bar{\gamma}) \cdot \frac{\pi(t_1)}{E(t_1)}$. This means that, if fund $i$ is still solvent, then $\pi_i(t_2) = \pi_i(t_1) \times \bar{\gamma}_i(t_1)/E_i(t_1)$, i.e. the change in share price is proportional to the change in equity. On the other hand, if $i$ is in default then $\pi_i(t_2) = 0$. Thus, the new totals of redeemable assets are

$$R(t_2) \cdot \tau_n = R(t_1) \cdot \text{Diag}(\bar{\gamma}) \cdot \text{Diag} \left( \frac{\bar{\gamma}}{E(t_1)} \right) \cdot \tau_n = R(t_1) \cdot \text{Diag}(\bar{\gamma}) \cdot \text{Diag} \left( \frac{E(t_1)}{E(t_1)} \right)^{-1} \cdot \bar{\gamma}.$$

Then, by applying equation (1) at $t_2$, and plugging in the previous results we obtain,

$$\bar{\gamma}(\gamma) = \left[ R(t_1) \cdot \text{Diag} \left( \frac{\bar{\gamma}}{E(t_1)} \right) \right] \cdot \bar{\gamma} + A(t_2) \cdot \tau_m + C(t_2) + B - L,$$

which, by rearranging, leads to

$$X_{\gamma} \cdot \bar{\gamma}(\gamma) = A(t_2) \cdot \tau_m + C(t_2) + B - L. \tag{19}$$

and we can conclude as $X_{\gamma}$ is supposed nonsingular.

A.2 Proof of non-singularity with no redemptions

We prove in this section proposition 2, meaning that the result of proposition 1 are applicable to a system of funds verifying the necessary conditions, and we discuss equivalent characterisations. We denote simply $X$ without specifying a set of defaults, as all arguments are the same if we replace the initial matrix $R(t_1)$ of cross-holdings by $R(t_1) \cdot \text{Diag}(\bar{\gamma})$.

Proposition 4. If no fund in the system is completely owned by other funds in it, then $X$ is nonsingular.

Proof. We will show that $Y := X^T$ is a strictly diagonally dominant matrix. First, let us observe that funds do not hold their own shares, hence $\forall i$, $Y_{i,i} = X_{i,i} = 1$. Second, for all $i, j$ such that $i \neq j$, $Y_{i,j} = X_{j,i} = -R_{j,i}(t_1)/E_i(t_1)$, which gives

$$\forall i \in \{1, \ldots, n\}, \quad \sum_{j \neq i} |Y_{i,j}| = \frac{1}{E_i(t_1)} \sum_{j} R_{j,i}(t_1).$$

Then, given that $R$ is the matrix of cross-holdings, evolving based on the NAV of the funds held, our assumption that no fund is totally held by other funds means that $\forall i, E_{i}(t_1) > \sum_j R_{j,i}(t_1)$. Thus, we have $\forall i, |Y_{i,i}| = 1 > \sum_{j \neq i} |Y_{i,j}|$, i.e. $Y$ is strictly diagonally dominant. From the Levy–Desplanques theorem we conclude that $Y$ is invertible. Therefore $X$ is invertible too.

In practice though, the condition of proposition 4 is not met in our network of funds. Thus, we give an improved version of the proposition, which generalizes the previous.

Proposition 5. If every fund in $\mathcal{F}$ admits an ultimate external investor, then $X$ is nonsingular.
This condition means that, given a fund $u \in \mathcal{F}$,
- either it admits directly an external investor, i.e. one that does not belong to $\mathcal{F}$, such that $E_u > \sum_j R_{j,u}$;
- or there exists a path $v_1 \rightarrow \ldots \rightarrow v_n \rightarrow u$ in $\mathcal{F}$, where $i \rightarrow j$ means that $i$ holds shares of $j$, such that $v_1$ has external investors.

**Proof.** Let us denote $Y := X^T$ as in the preceding proof. We can show that $Y$ is a *weakly chained diagonally dominant matrix* (WCDD), which is more general and encompasses the case of a strictly dominant diagonal. First, the system of funds verifies $\forall u \in \mathcal{F}, E_u \geq \sum_j R_{j,u}$, which implies that $Y$ is weakly diagonally dominant, following the previous calculus. Second, the hypothesis of the proposition ensures that, in the reversed graph that corresponds to $Y$, if there is no strict diagonal dominance of a row $u$, it is possible to find a path $u \rightarrow v_1 \rightarrow \ldots \rightarrow v_n$ where there is strict diagonal dominance in $v_n$. These two conditions define $Y$ as a WCDD matrix. Shivakumar and Chew (1974) show that all such matrices are nonsingular, which concludes the proof. ■

**Corollary 5.1.** If no cycle exists in the inter-funds cross-holdings, then $X$ is nonsingular.

**Proof.** In that case $R$ is a weighted DAG (directed acyclic graph), where condition (ii) of proposition 5 is verified by assumption. Then we know that we can obtain a topological order $\prec$ of the nodes (Cormen et al., 2009), i.e. the funds. It verifies $i \rightarrow j \implies i \prec j$, where $i \rightarrow j$ denotes that there exists an edge from $i$ to $j$, i.e. $i$ owns shares of $j$. Then, we can find a fund $i$ that is minimal for $\prec$, meaning that it has no investors within the set of funds. Therefore, as $E_i > 0$ by assumption, it must have external investors, verifying condition (i).  

This last case corresponds to the situation where the step-wise computation of Chrétien et al. (2020) ends in finite time. Equipped with proposition 5 we can now prove our main result.

**Proof of proposition 2.** First, suppose that there is a set $\mathcal{N} \subseteq \mathcal{F}$ so that each fund in $\mathcal{N}$ is fully owned by other funds in the group. We can assume $\mathcal{N} = \{1, \ldots, m\}$, as this corresponds to a simple reordering of the rows of $X$, which is rank-preserving. Thus we have

$$\forall j \in \mathcal{N}, \sum_{i \in \mathcal{N}} R_{i,j} E_j = \sum_{i=1}^m R_{i,j} E_j = 1 \quad (20)$$

Let $M = (R_{i,j}/E_j)_{1 \leq i,j \leq m}$. Then, by equation (20) we have $\iota_m^T \cdot M = \iota_m^T$, meaning that 1 is an eigenvalue of $M^T$. Therefore, 1 is also an eigenvalue of $M$, so that there exists $\eta \in \mathbb{R}^m$ such that $M \cdot \eta = \eta$. Then, let $M = R(t_1) \cdot \text{Diag}(E(t_1))^{-1} = I_n - X$, and let $\hat{\eta} = (\eta_1 \cdots \eta_m 0 \cdots 0)^T \in \mathbb{R}_n$. Then, we can write

$$M = \begin{pmatrix} M & K \\ 0_{n-m,m} & L \end{pmatrix}$$

where $K \in \mathbb{R}^{m \times (n-m)}$, $L \in \mathbb{R}^{(n-m) \times (n-m)}$ and $0_{n-m,m}$ is the null matrix of size $(n-m) \times m$. Thus, we get $M \cdot \hat{\eta} = \hat{\eta}$, i.e. $\hat{\eta}$ is an eigenvector of $M$ with eigenvalue 1. It follows that $X \cdot \eta = 0$. Therefore, $X$ is singular.

Second, assuming that no such set exists, we show that the system verifies the conditions of proposition 5. Suppose by contradiction that there exists a fund $i$ with no ultimate external investor. Then, denote by $\mathcal{U}$ the set of all ultimate investors of $i$. Let $j \in \mathcal{U}$, then $j$ has all its investors in

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Note that the proof could proceed from the topological order without relying on proposition 5: if we reorder funds according to the topological order, we obtain a new matrix $R^T$ that is upper triangular with zero diagonal. This is a rank preserving transformation as we simply permute rows. It follows that, when applying the same transformation to $X$, the resulting matrix is upper triangular with ones on the diagonal. This implies that $X$ is nonsingular.
Indeed, no external investor is possible, as this would be an ultimate external investor of \( i \) too. Moreover, for any investor \( k \) of \( j \), we have an edge \( k \rightarrow j \) in the funds graph, meaning that \( k \rightarrow j \rightarrow i \) is a valid path, and thus \( k \in \mathcal{U} \). We have shown that \( \mathcal{U} \) defines a set where each fund is completely owned by other funds within it, which contradicts our assumption. We conclude that all funds admit ultimate external investors, and as a consequence that \( X \) is nonsingular.

The same result can be obtained from the perspective of spectral graph theory. Then, \( X \) can be identified as a strict generalized random walk Laplacian in the sense of Veerman and Lyons (2020), meaning a matrix of the form \( I_n - a \cdot S \) where \( a \in \mathbb{R}^n \), with \( a < I_n \) and \( S \in \mathbb{R}^{n \times n} \) is a (row) stochastic matrix, corresponding to the normalized adjacency matrix of the graph of the fund system.

### A.3 Determining defaults

The approach presented here is similar to papers in the strand of Eisenberg and Noe (2001), with the exception that we have cross-holdings, not liabilities, i.e. a stock approach instead of flows and clearing payment between agents. In this it is more similar to Barucca et al. (2020), except that our underlying mathematical argument and relies on the contraction principle instead of the Knaster–Tarski theorem.

Let \( M = A(t_2) \cdot \epsilon_m + C(t_2) + B - L \). We define \( \Phi: \mathbb{R}^n \rightarrow \mathbb{R}^n \) the mapping given by

\[
\Phi(\bar{E}) := \left( R(t_1) \cdot \text{Diag} \left( E(t_1) \right) \right)^{-1} \cdot (\bar{E} \lor 0) + M .
\]

We show that this mapping is a contraction for \( \| \cdot \|_2 \) under assumption \( H_\emptyset \). Let \( e_1, e_2 \in \mathbb{R}^n \), then

\[
\| \Phi(e_1) - \Phi(e_2) \|_2 = \left\| R(t_1) \cdot \text{Diag} \left( E(t_1) \right) \right\|^{-1} \cdot (e_1 \lor 0 - e_2 \lor 0) + M - M \leq \left\| R(t_1) \cdot \text{Diag} \left( E(t_1) \right) \right\|^{-1} \cdot \| e_1 - e_2 \|_2
\]

Then, we know that \( R(t_1) \cdot \text{Diag} \left( E(t_1) \right) \) is the transpose of a substochastic matrix, so that its norm will be of 1 maximum, and is equal to its largest eigenvalue. Moreover, the invertibility of \( X_\emptyset \) that we have proved in \( \text{A.2} \) under assumption \( H_\emptyset \) implies precisely that 1 is not in the spectral radius of \( R(t_1) \cdot \text{Diag} \left( E(t_1) \right) \). Therefore, its largest eigenvalue is strictly less than 1. This is enough to conclude that \( \| \Phi(e_1) - \Phi(e_2) \|_2 < \| e_1 - e_2 \|_2 \), i.e. \( \Phi \) is a contraction.

Then, given that \( \mathbb{R}^n \) is a complete metric space we can conclude from the contraction principle that \( \Phi \) admits a unique fixed point (Subrahmanyam, 2018). Then, denoting as \( E(t_2) \) this fixed point we can define \( \gamma^* = \{ i \in \mathcal{F} : E(t_2) \leq 0 \} \), and it comes that \( \bar{E}(\gamma^*) = E(t_2) \).

### B Data supplements
**FIGURE 14**: Decomposition of climate data along dimensions of interest.

Left panel: carbon emissions on scopes 1 and 2 (in log scale on the x axis) by sectors of firms (on the y axis). Only sectors with more than 50 entities are kept. Additionally, data analysis not presented here shows that there are also marked differences between countries with regard to carbon intensity as some of them concentrate a larger number of polluting industries. Right panel: some important variations between countries (on the y axis) are observed with regard to climate physical risk exposure (on the x axis). Only countries with more than 100 entities are kept. The blue dashed line represents the euro area average.

Sources: Urgentem, 427 and authors’ calculations.
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(a) Carbon intensity categorisation  
(b) Physical risk categorisation

**Figure 15:** Grouping into deciles of risk as used in the simulation framework.

Left panel: all funds whose securities with available carbon ratings represent more than 50% of their portfolio by market value distributed into different deciles. The allocation is based on the fund-level CWA coefficient defined by equation (14). Category 1 represents the greenest 10% of classified funds, category 2 the 10% to 20% greenest, etc, and NC includes all non-classified funds (falling below the 50% threshold). The column with mean CWA is an unweighted average across all funds in a category. Note that, although all securities have a positive carbon emissions associated, some funds may have a negative weighted average because they short polluting securities.

Right panel: all funds whose securities with available climate physical risk exposure represent more than 20% of their portfolio by market value distributed into different deciles. The allocation is based on the fund-level PR coefficient defined by equation (15). Category 1 represents the greenest 10% of classified funds, category 2 the 10% to 20% greenest, etc, and NC includes all non-classified funds (falling below the 20% threshold). The column with mean PR is an unweighted average across all funds in a category.

Source: authors’ computations.
Figure 16: Quantification of the bias of financial markets and investment funds for riskier assets on the transition risk and physical risk dimensions.

The unweighted average is taken as a simple mean of the carbon intensity or physical risk index respectively. These two variables are the ones defined in section 3. The value is constant as we generally do not have full time series available for these data, and we use the last value known for the sub-sample of firms where different points in time are available. The “Funds average” value is obtained as a weighted average of the climate variables, where the weights are given by the proportion of securities in the aggregated holdings of all funds in our sample. Likewise, the “Market average” is obtained as a weighted average of the respective climate variables where weights used are the total market valuations of assets.

We observe in particular that the carbon intensity of both markets and investment funds has been decreasing on average over the last three years. On the other hand, both exposures to physical risk have been slightly increasing (which reflects portfolio choices as the physical risk values of securities are fixed here).

Source: CSSDB, Urgentem, 427, Refinitiv and authors’ computations.
C Results supplements

C.1 Complementary result figures

Figure 17: Bi-dimensional histogram of the market shocks used in 4.3 and 4.4, with a decomposition between asset types. The x-axis gives the carbon intensity of the assets while the y-axis gives the shocked returns. Source: authors’ calculations.
(a) Transition risk market shock

(b) Physical risk market shock

**Figure 18**: Time series of indirect severity.
Left panel: indirect severity for different months, based on the same transition risk market shock that is used in section 4.3, and represented in figure 17a. Right panel: indirect severity for different months, based on the same physical risk market shock that is used in section 4.4, and represented in figure 17b. For both, the indirect severity is defined in section 4.1 relative to a uniform shock with similar aggregate direct impact. Source: authors’ calculations.